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Original research article

Dynamic analysis of reinforced concrete plate on elastic foundation using state space method and refined plate theory

Nguyen Trong Hiep [a,*]

[a] *Department of Highway and Airport, Faculty of Civil Engineering, University of Transport and Communications, Ha Noi, Vietnam*

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ABSTRACT

In rigid pavement analysis, along with domination of finite element method, the analytical approach with close form solutions is still very valuable for development of new methodologies, served as basis for practical calculation. Among multiple approaches, the state space method and refined plate theory are relevant for slab and foundation interaction analysis. In this paper, the dynamic response of rectangular reinforced concrete plates on an elastic foundation is developed based on state space method. The system of differential equations of plate vibration is developed using Hamilton's principle. The displacements are represented through a double trigonometric series for the simply-supported rectangular plate. The state space method is used to find the plate vibrational responses. A numerical example is used to investigate the displacement at the center of the plate according to different structural conditions of foundation and applied load, thus evaluating the influence of these parameters on displacement.

1. Introduction

In transportation infrastructure construction, Portland cement concrete (PCC) pavement is commonly applied in reality for adverse performance and weather conditions due to its advanced properties of high stiffness, water stability, thermal stability, high abrasive resistance, and long-life service. PCC pavement is commonly used for heavy loading and/or high traffic density highway routes. For airports, rigid or PCC pavement is also an appropriate alternative for airfield construction components of runway, taxiway, apron, etc, which serve to increase wheel load of the new generation of civil aircraft. Therefore, PCC has been dominated by the type of pavement applied for construction.

The typical rigid pavement structures comprise of PCC slab placing on base/subbase layers of bound and/or unbound granular, usually called foundation, and subgrade soil. Although these are separate components with different properties, the slab and foundation works together as a unified system.

In structural construction, the dynamics of beam, plate have been studied for decades. In order to describe the performance of pavement systems, various foundation models have been proposed. The simplest model is the Winkler model for elastic foundations, springs of constant stiffness. Pasternak [1] proposed a new model by considering shear interactions between the plate and the background. In addition, more efficient foundation models have been proposed, for example, the improved Pasternak Model [2] for reinforced soils. With the analytic or semi-analytic approach, the Navier solution is used for the simple linked rectangular plate [3], [4], and for the circular plate, the Ritz method [5], the functional method can be used. Green [6]. T. D. Hien [7] calculated the plate on an irregular elastic foundation by the Ritz method. Some domestic studies on plates in general and plate dynamics, such as Trung et al., have developed a 3-node smooth finite element method for the problem of composite panels [8] and Mindlin plates [9] on the viscous foundation subjected to moving loads.

* Corresponding author.
E-mail address: nguyentronghiep@utc.edu.vn. (N Trong Hiep)
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Nguyen Van Loi and Tran Minh Tu [10] calculated the specific vibration of layered composite panels according to the theory of high-order shear strain with eight displacement unknowns. Nguyen Tri Dung and Dang Sy Lam [11] calculated the variable mechanical properties of composite panels by the 9-node finite element method. T. D. Hien [12] calculated the vibrations of functionally graded plates resting on elastic foundations using the state space method.

In the world, there are also some authors interested in the calculation of cement concrete pavement slabs. Cai Jing et al. [13] calculated the vibration of the concrete pavement of the cement concrete pavement on the viscoelastic foundation by analytical method. Taheri et al. [14] calculated a cementitious concrete slab on a viscoelastic foundation subjected to the mobile live load of an aircraft by the finite element method. Xiao Tian et al. [15] using ANSYS software analyzed plates on elastic foundations with different speeds of vehicle dynamics. Shimpi and Patel [16] used the refined plate theory to analyze orthotropic plates. D. T. Thuy et al. [17] derived the analytical solution for the dynamics of functionally graded plate laying on viscoelastic foundation

Among multiple approaches for rigid pavement calculation and design, along with numerical methods represented with well-known finite element methods, the analytical methods with close-form solution are still valuable and attract more and more scholars. These are solid background for new structural computation methodologies, which enable to diversify alternatives for practical calculations. The article performs the vibration of reinforced concrete plates on elastic foundations by state space method, a variation based on the studies of T. D. Hien. [12] and D. T. Thuy et al. [17]. The displacement of the plate is represented through a double trigonometric series as the basis for finding the vibrational response by analytic solution for a single quasi-rectangular plate.

2. Material and Methods

a. Differential equation of vibration of reinforced concrete plate

Model of reinforced concrete slab dimension a x b x h laying on elastic foundation is described in Figure 1.

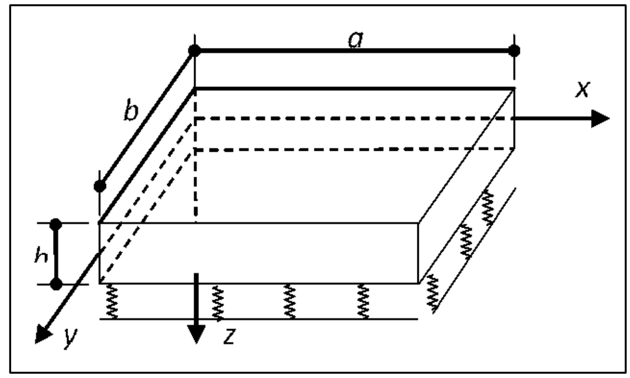

Fig. 1. Plate model on elastic foundation.

When the plate is subjected to load, concrete cracks will reduce the stiffness of the slab. The reinforcement having high strength will prevent cracks in the concrete. In this study, it is assumed that the displacement and deformation of plate is continuous. The reinforced concrete plate is also assumed to be a homogeneous plate with the elastic modulus E_c of cement concrete. The elastic foundation is assumed to be a Winkler foundation with the stiffness of the foundation K_n. Applying the Shimpi's high-order shear strain assumption [16], the displacement components at coordinates (x, y, z) of the plate are presented in Eq. (1) as follows:

$$W(x,y,z,t)=w_b(x,y,t)+w_s(x,y,t)$$
$$U(x,y,z,t)=-z\frac{\partial w_b}{\partial x}+\frac{1}{4}z\frac{\partial w_s}{\partial x}-\frac{5}{3}z\left(\frac{z}{h}\right)^2\frac{\partial w_s}{\partial x}$$
$$V(x,y,z,t)=-z\frac{\partial w_b}{\partial y}+\frac{1}{4}z\frac{\partial w_s}{\partial y}-\frac{5}{3}z\left(\frac{z}{h}\right)^2\frac{\partial w_s}{\partial y} \qquad (1)$$

Where (w_b, w_s) is the displacement components at the middle plane of the plate. The equation of the deformation is found by derivating the system of Eq. (1) and presented in Eq. (2) as follows:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=z\begin{Bmatrix}\kappa_x^b\\ \kappa_y^b\\ \kappa_{xy}^b\end{Bmatrix}+f\begin{Bmatrix}\kappa_x^s\\ \kappa_y^s\\ \kappa_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \qquad (2)$$

Where:

$$\begin{Bmatrix} \kappa_x^b \\ \kappa_y^b \\ \kappa_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \kappa_x^s \\ \kappa_y^s \\ \kappa_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix} \quad (3)$$

$$f(z) = -\frac{1}{4}z + \frac{5}{3}z\left(\frac{z}{h}\right)^2, \; g(z) = 5\left[\frac{1}{4} - \left(\frac{z}{h}\right)^2\right] \quad (4)$$

Apply Hamilton's principle, the vibration equations of the plate are presented as follows:

$$\begin{aligned} &\frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - K_w(w_b + w_s) - K_c(\dot{w}_b + \dot{w}_s) + f \\ &\quad = I_0(\ddot{w}_b + \ddot{w}_s) + I_1\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - I_2 \nabla^2 \ddot{w}_b - J_2 \nabla^2 \ddot{w}_s \\ &\frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} - K_w(w_b + w_s) - K_c(\dot{w}_b + \dot{w}_s) + f \\ &\quad = I_0(\ddot{w}_b + \ddot{w}_s) + J_1\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - J_2 \nabla^2 \ddot{w}_b - K_2 \nabla^2 \ddot{w}_s \end{aligned} \quad (5)$$

Where:

$$\begin{aligned} I_i &= \int_{-h/2}^{h/2} \rho(z) z^i dz, \\ J_i &= -\frac{1}{4} I_i + \frac{5}{3h^2} I_{i+2} \\ K_2 &= \frac{1}{16} I_1 - \frac{5}{6h^2} I_4 + \frac{25}{9h^4} I_6 \\ \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \end{aligned} \quad (6)$$

b. Vibration of plates using state-space method

Apply Navier approach to derive the closed-form solutions of displacement function into double trigonometric series. The sinusoidal function satisfying all the boundary conditions can be presented as follows:

$$\begin{aligned} w_b(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\{W_{bmn}(t)\sin\alpha x \sin\beta y\} \\ w_s(x,y,t) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\{W_{smn}(t)\sin\alpha x \sin\beta y\} \end{aligned} \quad (7)$$

Where:

$$\alpha = \frac{m\pi}{a}, \beta = \frac{n\pi}{b} \quad (8)$$

Substitute the system of equations Eq. (7) into the equation Eq. (5), the system of equations for the plate vibration is represented as follows:

$$\begin{bmatrix} s_{33} & s_{34} \\ s_{34} & s_{44} \end{bmatrix} \begin{Bmatrix} W_{bmn} \\ W_{smn} \end{Bmatrix} + \begin{bmatrix} m_{33} & m_{34} \\ m_{43} & m_{44} \end{bmatrix} \begin{Bmatrix} \ddot{W}_{bmn} \\ \ddot{W}_{smn} \end{Bmatrix} = \begin{Bmatrix} F_{mn} \\ F_{mn} \end{Bmatrix} \quad (9)$$

Where the terms F_{mn} that are the loading vector, which can be presented as follows:

$$F_{mn} = \frac{4}{ab}\int_0^b\int_0^a q(x,y,z,t)\sin\alpha x \sin\beta y dx dy \quad (10)$$

For uniform and sinusoidal distributed loads:

$$q(x,y,z,t) = \begin{cases} q_0 \hat{F}(t) & \text{: for uniform distributed load} \\ q_0 \sin\dfrac{\pi x}{a}\sin\dfrac{\pi y}{b}\hat{F}(t) & \text{: for sinusoidal distributed load} \end{cases} \quad (11)$$

Substituting Eq. (11) into Eq. (10), we have:

$$F_{mn} = \begin{cases} \dfrac{16 q_0}{mn\pi^2}\hat{F}(t) & \text{: for uniform distributed load} \\ q_0 \hat{F}(t) & \text{: for sinusoidal distributed load} \end{cases} \quad (12)$$

Consider the three common cases of the loading function $\hat{F}(t)$ by time, we can get the loading vectors as follows:

Step load: when time t in the range of [0, t1], the value of loading vector is 1 or maximum magnitude of loading vector. From the time t greater than t1, the loading vector will be suddenly turn to zero.

$$\hat{F}(t) = \begin{cases} 1 & 0 \le t \le t_1 \\ 0 & t \ge t_1 \end{cases} \quad (13)$$

Triangular load: the mechanism of applying load will be linearly increased from 0 to 1 or maximum magnitude of loading vector. From the time t greater than t1, the loading vector will be suddenly turn to zero.

$$\hat{F}(t) = \begin{cases} t/t_1 & 0 \le t \le t_1 \\ 0 & t \ge t_1 \end{cases} \quad (14)$$

Sinusoid load: when time t in the range of [0, t1], the value of loading vector will be follow the sinusoid period 1 or maximum magnitude of loading vector. From the time t greater than t1, the loading vector will be suddenly turn to zero.

$$\widehat{F}(t) = \begin{cases} \sin(\pi t/t_1) & 0 \le t \le t_1 \\ 0 & t \ge t_1 \end{cases} \tag{15}$$

The plate vibration presented in equation Eq. (9) can be rewritten as follows:

$$\dot{\boldsymbol{Z}} = \boldsymbol{AZ} + \boldsymbol{b} \tag{16}$$

Where

$$\boldsymbol{Z} = \begin{Bmatrix} W_{bmn} \\ W_{smn} \\ \dot{W}_{bmn} \\ \dot{W}_{smn} \end{Bmatrix}; \boldsymbol{b} = \begin{Bmatrix} 0 \\ 0 \\ b_1 \\ b_2 \end{Bmatrix} \tag{17}$$

with components

$$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \boldsymbol{M}^{-1}\boldsymbol{F}^* \tag{18}$$

The matrix A are rewritten as in Eq. (19) as follows:

$$\boldsymbol{A} = \begin{bmatrix} 0 & I \\ -\boldsymbol{M}^{-1}\boldsymbol{S} & 0 \end{bmatrix} \tag{19}$$

Solution of the system of differential equations of slab vibration presented in Eq. (19) as following:

$$\dot{\boldsymbol{Z}}(t) = e^{\boldsymbol{A}(t-t_0)}\boldsymbol{Z}(t_0) + \int_{t_0}^{t} e^{\boldsymbol{A}(t-\tau)}\boldsymbol{b}(\tau)\,d\tau \tag{20}$$

The initial displacement vector $\boldsymbol{Z}(t_0)$, and the exponential matrix $e^{\boldsymbol{A}(t-t_0)}$ are formed from the eigenvalues and eigenvectors of the matrix A.

c. Application example

In order to check the computation of the proposed method, and to investigate the influence of foundation stiffness on plate vibrations, an example with numerical was conducted with a real numerical data set.

The assumptions for computation is that a rectangular reinforced concrete slab with dimensions of length a, width b and thickness h.; the ratio width over thickness was set of a/h = 20 to ensure the assumption of thin plate. The main physical mechanical properties of elastic modulus (E_c), specific density (ρ_c) and deformation Poisson's ratio (ν) of PCC slab was set as common PCC and summaried in Table 1.

Table 1
Parameters of reinforced concrete slab.

Parameters	Denote	Value	Remark
Slab dimensions	a x b	3m x 4m	Rectangle
Concrete modulus	E_c	28 GPa	
Density	ρ_c	2,400 kg/m3	
Poisson's ratio	ν	0.3	

For convenience, the equivalent stiffness of foundation is rewritten as in Eq. (20):

$$k_w = \frac{K_w a^4}{D_c} \tag{20}$$

In that the flexural rigidity of slab was presented in Eq. (21):

$$D_c = \frac{E_c h^3}{12(1-\nu^2)} \tag{21}$$

In this calculation example, the foundation stiffness (k_w) and the maximum uniform load (q) applied on the slab surface were presented in Table 2.

Table 2
Parameters of foundation rigidity and loading vector.

Parameters	Denote	Value	Remark
Foundation stiffness	k_w	0; 1000	
Max. uniform load	q	105 N/m2	

The applied load for computation is uniform load, which will assumped to be be distributed equally on the plate surface. The magnitude of load will be varying by time according to applying rules of step, sinusoidal and triangular.

3. Result and Discussion

The results from the calculation example with input data set of concrete slab, foundation and applied load are represented in the graphs shown in Figure 2-4.

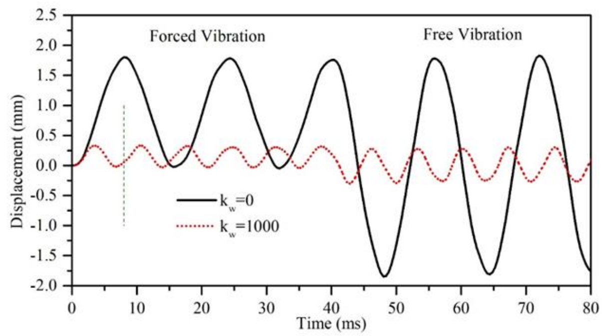

Fig. 2. The transient displacement at center of concrete slab due to step load.

The Figure 2 represent the vibration of the slab under the impact of step load applied on the slab surface. The amplitude of displace under the forced vibration is smaller than that of free vîbration. In the forced vibration, the displacements have positive values, while under free vibrations having positive and negative displacements. Fof the influence of foundation stiffness, when the stiffness increases, the displacement of plate reduce significantly.

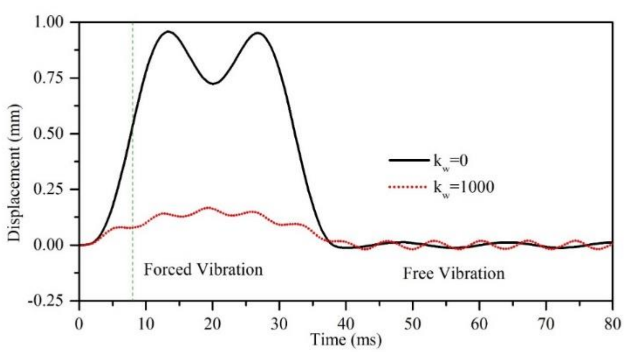

Fig. 3. The transient displacement at center of plate due to sinusoidal load.

The Figure 3 presents the transient displacement at center of plate under impact of sinusoidal load. In the forced vibration phase, the magnitude of plate displacements is significantly different between the two foundation stiffness values.

When changing to free vibration phase, the displacements are minor for both kw = 0 and kw = 1000, and the magnitude differences are also very minor. However, the vibration frequencies with kw = 0 is denser than that with kw = 1000.

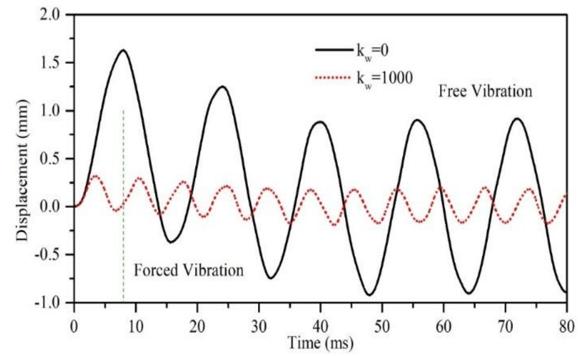

Fig. 4. The transient displacement at center of plate due to triangular load.

For the case that plate under triangular load, in the forced vibration phase, the displacements start at 0, and then steadily changing to state where the displacements having both values positive and negative in the next loading cycles. In free vibration phase, the displacements of the plate are uniformly vibrating around the 0 point.

4. Conclusions and Recommendations

The paper uses the state space method to calculate the forced and free vibrations for reinforced concrete plates on an elastic foundation. Vibration response of reinforced concrete plate using analytical method with displacement representation is a double trigonometric series according to Navier's approach. Dynamic responses are calculated for both forced and free vibration phases.

In order to validate the proposed method, a numerical example with assummed parameters represent for PCC material of plate. The foundation stiffness is also set at 0 and 1000 for investigation. The displacement at the center of the plate is investigate with uniform distributed load changing by time according to step, sinusoidal and triangular rules. The computation results outputed from the proposed method are evidently appropriate with structural common senses that the more stiffness of foundation, the less concrete displacement. It demonstrates the significantl influence of the foundation stiffness on the displacement of the plate.

In overall, the study presents a new approach for computation of structural plate on elastic foundation. This work has important meaning for calculation of structural engineering, and the results can be used in structural design of airport rigid pavement.

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